# APPLICATION OF QUEUING MODELS IN ENHANCEMENT OF BANK SERVICES AT AFRAM COMMUNITY BANK LIMITED (MAAME-KROBO BRANCH) 

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$\overline{\text { Abstract - Queuing problems is an important problem to our everyday life. In general, customers are not willing }}$ to wait for long before services are rendered.

The analysis of the queuing model $(\mathrm{m} / \mathrm{m} / 1)$ shows that, if the number of servers is increased to two or more, the highest utilization factor of $82.57 \%$ for $\mathrm{m} / \mathrm{m} / 1$ will be decreased to $41.29 \%$ for $\mathrm{m} / \mathrm{m} / 2$ and a more decreased to $27.52 \%$ for an $\mathrm{m} / \mathrm{m} / 3$ model. That is, the more the servers, the lesser the busy time of the servers, the lesser the time customers spend in the queue and in the system and finally the lesser the number of customers in the system and in the queue.

Moreover, the results suggest that, the network system and service charges are to be worked on by management to improve on the services delivered to increase customers' satisfactions. The analysis further indicates that, customers are preferred to be in the banking hall for a maximum time period of 5-10 minutes

Keywords: Queues, Queuing models, Services.

## Introduction and background of study

In other to maintain peace, discipline and to avoid congestion at service centers, queues are used to achieve these purposes. Queues or waiting lines are very common to our everyday life. These waiting lines can be observed in the Banks, Hospitals, Supermarkets, Telephone exchange, etc. The first problems of queuing theory were raised by telephone calls.
A.K Erlang (1909) was the first to treat congestion problems in the beginning of the $20^{\text {th }}$ century. Queuing theory is a branch of mathematics that studies and models the act of waiting in lines in order to be served.

Queuing theory known by various other names such as Congestion theory, Mass service theory and Traffic theory (Copper, 1981) is a mathematically based technique for analyzing waiting lines (queues). In queuing theory, queuing models are used to estimate a real queuing situation, so that the queuing behavior can analyzed mathematically. Customers arriving at a service facility, waiting in a queue (line) if all servers are busy, then receiving services and finally departing from the service facility when he/she has been served is also term as queuing system. Queuing system is based on a set of customers, a set of server(s) and the order whereby these customers arrived and are given service or processed.

Whenever the demand for a service exceeds its supply, then queues are formed. Long waiting time in Banking is considered an indicator of poor quality and need improvement. Managing waiting lines create a great problem for managers who are seeking to improve upon quality bank services and customer satisfaction.

Banking is a business activity of accepting and safeguarding money owned by other individuals and entities and then lending out this money in other to earn a profit. And a Bank as defined by the Encarta World Dictionary
refers to a business that keeps money for individuals or companies, exchange currencies, give loans and offer other financial services.

In 1976, the Ghana Government through the Bank of Ghana (BoG) established rural and community Banks to channel credit to productive rural ventures and promote Rural Development. The underlining aim was to make credit institutions available to help promote growth and development in our rural areas. As at April 2016, there are one hundred and forty (140) licensed Rural and Community Banks in Ghana. The other queuing disciplines are the Last Come First Served (LCFS) and Service in Random Order (SIRO).

Another factor that has an important effect on the behavior of a queuing system is the method that customers use to determine which line to join. In this study, customers are bound to join a single queue or line due to the existence of a single Server or Teller. In other Banks where there are several Servers or Tellers, customers have the option to choose or join the line they prefer.

However, this study is designed to help the management of Afram Community Bank Limited about Employer efficiency and also help reduce customers waiting time in the Banking Hall. Most customers are not comfortable waiting or queuing (Olaniyi, 2004). According to Elegalam (1978), customers are not prepared to spend cost on queuing. The time wasted on the queue would have been judiciously used elsewhere (the opportunity cost of time spent in queuing).

Notwithstanding that, despite all the efforts made by Banks, queues continues to exist in Banking Halls (Mbuvi, 2013). The existence of long queues in the banking halls for several hours leads to waste of time, limit productivity and create less patronage. Since banks plays a vital role in the Economic Development of the country (Ghana), a slight decline in performance affects the country's economy at large.

Before the introduction of banking system in Ghana, there was already an existence of Mathematical models which were used for Forecasting or predicting the consequences or outcome of queues (waiting lines) in banks. During an analysis on a formal data I carried out at the Afram Community Bank Limited on a sample of customers. It was revealed that most or majority of the customers complained of being in the queue for long due to the following problems below;
i. Network problem in the banking hall
ii. The existence of only one Server or Teller
iii. High bank service charges on customers

However, on the network problem, customers are obliged to be in the queue until the network is available for service to start. Also, with the event of high service charges which was mention as one of the problems in the Afram Community Bank which leads to long queue or keep customers waiting implies that, some customers who goes to the service desk sometimes spent much time trying to know how and where these charges are emerging from. By this, keeping the other customers in the queue for long. And the other factor which was the single server or teller issue also implies that, all customers must pass through the same server even when the queue is very long, before they can receive services. Hence, due to these findings obtained from the customers, it has become very prudent to use a mathematical model to check or analyze and reduce the queuing problems in Afram Community Bank Limited in order to enhance quality bank services and increase customer satisfaction.

Base on this study, these research questions helps to identify the major drawback of long queues or waiting lines in the Afram Community Bank Limited. It seeks to address the following questions;
i. Which services do customers perform most in the Afram Community Bank Limited?
ii. What major problems do customers normally encounter during services in the Afram Community Bank Limited?
iii. How much time do customers spend in the queue and in the system at the Afram Community Bank Limited?
iv. How do customers feel in the existence of only one server or teller in the Afram Community Bank Limited?
v. What is the attitude and performance of workers (Employees) to customers in the Afram Community Bank Limited?

The main focus of this study is to determine how effective single channel queuing model $(M / M / 1)$ is used to analyze the utilization factor and performance measure in the banking hall and how it relates to customer satisfaction.

## RESEARCH METHODS

## Test of Hypothesis:

$\mathrm{H}_{\mathrm{o}}$ : Customers are not satisfied with the $\mathrm{M} / \mathrm{M} / 1$ Queuing Model
$\mathrm{H}_{1}$ : Customers are satisfied with the $\mathrm{M} / \mathrm{M} / 1$ Queuing Model
Questionnaire data is analyzed by computing percentages of customer's responses to each question.

## Single-Channel Queuing Model with Poisson Arrivals and Exponential Service Time

Under this research, the single-channel queuing model is used because of the existence of only one server or teller in the banking hall of the Afram Community Bank Limited.

## Single service facility



Figure 1.0: Single channel queuing system

## Assumptions of the Model

We assume that the following conditions exist in this type of system.
i. Arrivals are served on a first-in-first-out (FIFO) basis and every arrival waits to be served regardless of the length of the queue.
ii. Arrivals are independent of preceding arrivals, but the average number of arrivals (arrival rate) does not change over time.
iii. Arrivals are described by a poison probability distribution and come from an infinite population.
iv. Service time varies from one customer to the next and is independent of one another, but their average rate is known.
v. Service times occur according to the negative exponential probability distribution.
vi. The service rate is faster than the arrival rate.

## Equations of Single-Channel Queuing Model

Let $\lambda=$ Mean number of arrivals per time period $=$ Mean arrival rate $=\frac{1}{E[r]}$
$\mu=$ Mean number of people or customers per time period $=$ Mean service rate $=\frac{1}{E[s]}$
i. $L_{s}=$ average number of units (customers) in the system

$$
\begin{equation*}
\mathrm{L}_{\mathrm{S}}=\frac{\lambda}{\mu-\lambda} \tag{3.2}
\end{equation*}
$$

ii. $\mathrm{W}_{\mathrm{s}}=$ average time a unit spends in the system (waiting time plus service time)

$$
\begin{equation*}
\mathrm{W}_{\mathrm{s}}=\frac{1}{\mu-\lambda} \tag{3.3}
\end{equation*}
$$

iii. $\mathrm{L}_{\mathrm{q}}=$ average number of units (customers) in the queue

$$
\begin{equation*}
\mathrm{L}_{\mathrm{q}}=\frac{\lambda^{2}}{\mu(\mu-\lambda)} \tag{3.4}
\end{equation*}
$$

iv. $\mathrm{W}_{\mathrm{q}}=$ average time a unit (customer) spends in waiting in the queue

$$
\begin{equation*}
\mathrm{W}_{\mathrm{q}}=\frac{\lambda}{\mu(\mu-\lambda)} \tag{3.5}
\end{equation*}
$$

$\mathrm{P}=$ utilization factor of the system.
$\varrho=\frac{\lambda}{\mu}$
$\varrho$ is the number of arrivals per mean service time.
If $\rho<1$, then steady state probability exists.
If $\rho>1$, the arrivals come at a faster rate than the server can accommodate.
When $\rho=1$, steady state does not occur.

## The Erlang Distribution

The Erlang distribution is used to model situations where the inter-arrival times do not appear to be exponential. It is a continuous random variable ( T ) whose density function is specialized by two parameters; a scale parameter, $\mu$, and a shape parameter k (where k is a positive integer).

A random variable T with density function $\mathrm{f}_{\mathrm{k}}$ is called Erlang distribution. When $\mathrm{k}=1$ the Erlang and exponential distributions coincide. Suppose that $T$ is the sum of the independent, exponential random variables $T_{1} \ldots, T_{k}$, each with parameter $\mu$. Then $T$ is Erlang distributed with density $f_{k}$, and its mean is
$\mathrm{E}(\mathrm{T})=\mathrm{E}\left(\mathrm{T}_{1}\right)+\ldots+\mathrm{E}\left(\mathrm{T}_{\mathrm{k}}\right)=\frac{k}{\mu}$
And its variance is also given as
$\operatorname{Var}(\mathrm{T})=\operatorname{Var}\left(\mathrm{T}_{1}\right)+\ldots+\operatorname{Var}\left(\mathrm{T}_{\mathrm{k}}\right)=\frac{k}{\mu^{2}}$

## Probability Distribution

Probability of the number of customers in the system is often used to describe the behavior of a queuing system by means of estimating the probability distribution or pattern of the arrival times between successive customer arrivals (inter arrival times). There are many well-known probability distributions which are of great importance in the world of probability such as Poisson, exponential, binomial, geometric distribution, and many others. However, we will discuss the important distributions which have been found useful for this study.

## Poisson Distribution

The Poisson distribution is used to determine the probability of a certain number of arrivals occurring at a given time with the simple model assumes that the number of arrivals occurring within a given interval of time $t$, follows a Poisson distribution with parameter $\lambda_{\mathrm{t}}$. The Poisson process is an extremely useful process for modeling purposes in many practical applications such as to model arrival process of queuing models.

In this model it is assume that only arrivals are allowed at a rate $\lambda$ per unit time. The assumption here is that, there is no one in the system at $t=0$. We usually assume and try to determine how many arrivals might enter the system within some time ( t ). The arrivals of the customers can be assumed to follow Poisson distribution. That is $\mathrm{n}(\mathrm{t})$ is the number of customers or arrivals that has occurred from time 0 to time t .
The Poisson distribution is given as;
$\mathrm{P}_{\mathrm{n}}(\mathrm{t})=\frac{(\lambda t)^{n}}{n!} e^{-\lambda t}$
Where

- t describes the period 0 to t .
- n is the total number of arrivals in the period 0 to $t$.
- $\quad \lambda$ is the total average arrival rate.

Poisson distribution is said to hold when;

$$
\begin{equation*}
\mathrm{E}(\mathrm{n})=\operatorname{Var}(\mathrm{n})=\lambda \tag{3.10}
\end{equation*}
$$

## Exponential Distribution

The most commonly used queuing models are based on the assumption of exponentially distributed service times and inter arrival times. A random variable $\mathbf{X}$ has an exponential distribution with parameter $\lambda$ if the density of $\mathbf{X}$ is given by
$\mathrm{f}(\mathrm{t})=\lambda \mathrm{e}^{-\lambda \mathrm{t}} \quad$ if $(\mathrm{t} \geq 0)$
Then
$\begin{aligned} & \mathrm{E}(\mathrm{X})=\frac{1}{\lambda} \\ & \operatorname{Var}(\mathrm{X})=\frac{1}{\lambda^{2}} \quad\end{aligned} \quad-$

The exponential distribution has the no-memory property. This means, for instance, that if inter-arrival times are exponentially distributed with rate or parameter $\lambda$, then no matter how long it has been since the last arrival, there is a probability $\lambda \Delta \mathrm{t}$ that an arrival will occur during the next $\Delta \mathrm{t}$ time units. Inter-arrival times are exponential with parameter $\lambda$ if and only if the number of arrivals to occur in an interval of length $t$ follows a Poisson distribution with parameter $\lambda$ t.

## Results and discussion

### 4.2 Queuing Analysis: Single Server (M/M/1) Model

Table 4.2.1 Statistics of mean inter-arrival time on $17^{\text {th }}$ march, 2018

| Statistic | Value |
| :--- | :--- |
| Sample Size | 19 |
| Range | 7 |
| Mean | 2.9474 |
| Variance | 6.0526 |
| Std. Deviation | 2.4602 |
| Coef. of Variation | 0.83471 |
| Excess Kurtosis | -1.5034 |

Source: field survey march, 2018
The above table shows the descriptive statistics of customers' inter-arrival time on $17^{\text {th }}$ march, 2018 in the Afram Community Bank Limited. It shows that, the mean inter-arrival time is 2.9474 . This mean value of 2.9474 can be used to calculate the mean arrival rate $(\lambda)$ using the formula for calculating lamda $(\lambda)$ as stated in chapter three.

Table 4.2.2 Analysis of customer's mean service time on $17^{\text {th }}$ march, 2018: Statistics of mean service time

| Statistic | Value |
| :--- | :--- |
| Sample Size | 14 |
| Range | 1 |
| Mean | 2.2857 |
| Variance | 0.21978 |
| Std. Deviation | 0.46881 |
| Coef. of Variation | 0.2051 |
| Std. Error | 0.12529 |
| Skewness | 1.0665 |
| Excess Kurtosis | -1.0341 |

Source: field survey march, 2018
The above table shows the results of customer's services time using Easy Fit software. The results show that, the mean service time is 2.2857 . This mean service time of 2.2857 can also be used to calculate the mean service rate $(\mu)$ using the formula in chapter for calculating $\mu$.

Table 4.2.3 Distributions of customer's inter-arrival time on 17th march 2018

| \# | Distribution | Parameters |
| :--- | :--- | :--- |
| $\mathbf{1}$ | Beta | $\square_{1}=0.04594 \square_{2}=0.17355: \mathrm{a}=8.5017 \mathrm{E} 16 \mathrm{~b}=7.0$ |
| $\mathbf{2}$ | Burr | $\mathrm{k}=3199.8 \square=2.2593 \square=160.73$ |
| $\mathbf{3}$ | Burr | $\mathrm{k}=2055.2 \square=2.2627 \square=131.59$ |
| $\mathbf{4}$ | Cauchy | $\square=1.9592 \square=2.8959$ |
| $\mathbf{5}$ | Chi-Squared | $\square=4$ |
| $\mathbf{6}$ | Chi-Squared | $\square=2$ |
| $\mathbf{7}$ | Dagum | $\mathrm{k}=0.00364 \square=668.18 \square=7.1015$ |
| $\mathbf{8}$ | Dagum | $\mathrm{k}=0.00473 \square=598.72 \square=7.0176$ |
| $\mathbf{9}$ | Erlang | $\mathrm{m}=3 \square=1.2105$ |
| $\mathbf{1 0}$ | Erlang | $\mathrm{m}=1 \square=2.0536$ |
| $\mathbf{1 1}$ | Error | $\mathrm{k}=100.0 \square=2.4602 \square=2.9474$ |
| $\mathbf{1 2}$ | Error Function | $\mathrm{h}=0.28742$ |
| $\mathbf{1 3}$ | Exponential | $\square=0.33929$ |
| $\mathbf{1 4}$ | Exponential (2P) | $\square=0.33929 \square=-1.0000 \mathrm{E}-14$ |
| $\mathbf{1 5}$ | Fatigue Life | $\square=0.55703 \square=3.3831$ |

From the table above, since the inter-arrival time follows an exponential distribution, the mean arrival rate $(\lambda)$ is shown on the table as 0.33929 . This mean arrival rate $(\lambda)$ of 0.33929 is used in the estimation of the parameters of the utilization factor and performance measure values.

Table 4.2.4 Distribution of customers mean service time on 17/03/2018

| \# | Distribution | Parameters |
| :--- | :--- | :--- |
| $\mathbf{1}$ | Beta | $\square_{1}=0.24714 ; \square_{2}=0.7002 \quad \mathrm{a}=2.0 \quad \mathrm{~b}=41.986$ |
| $\mathbf{2}$ | Burr | $\mathrm{k}=48.434 \square=1.0215 \mathrm{E}+6 \quad \square=2.0$ |
| $\mathbf{3}$ | Burr (4P) | $\mathrm{k}=0.75499 \square=1.2362: \square=8.9173 \mathrm{E}-6 \quad \square=2.0$ |
| $\mathbf{4}$ | Chi-Squared | $\square=2$ |
| $\mathbf{5}$ | Dagum | $\mathrm{k}=34.96 \square=39.984 \square=1.8312$ |
| $\mathbf{6}$ | Dagum (4P) | $\mathrm{k}=0.34236 \square=1.1842: \quad \square=0.16523 \quad \square=2.0$ |
| $\mathbf{7}$ | Erlang | $\mathrm{m}=23 \square=0.09615$ |
| $\mathbf{8}$ | Erlang (3P) | $\mathrm{m}=1 \square=0.2857 \square=2.0$ |
| $\mathbf{9}$ | Error | $\mathrm{k}=6.7122 \square=0.46881 \quad \square=2.2857$ |
| $\mathbf{1 0}$ | Error Function | $\mathrm{h}=1.5083$ |
| $\mathbf{1 1}$ | Exponential | $\square=0.4375$ |
| $\mathbf{1 2}$ | Exponential (2P) | $\square=3.5 \square=2.0$ |
| $\mathbf{1 3}$ | Fatigue Life | $\square=0.18365 \square=2.2478$ |

Since customer's service time follows an exponential distribution, the table shows that, the mean service rate ( $\mu$ ) is 0.4375 . This mean service rate of 0.4375 is used with the $\lambda$ value of 0.33929 in Table 4.2 .3 above for the estimation of the utilization factor and performance measures.

Table 4.2.5 Parameter Estimates on 17th march, 2018

| INPUT | TIME UNIT |
| :--- | :--- |
| Arrival rate $(\lambda)$ | 0.3393 |
| Service rate $(\mu)$ | 0.4375 |
| INTERMEDIATE CALCULATIONS | 2.9474 |
| Average inter- arrival time | 2.2857 |
| Average service time |  |
| PERFORMANCE MEASURES | 0.7755 |
| Average server Utilization (rho) | 0.2245 |
| Probability that the service unit is Idle (p) | 3.4552 customers |
| Average number of customers in the system $\left(\mathrm{L}_{\mathrm{s}}\right)$ | 2.6797 customers |
| Average number of customers in the queue $\left(\mathrm{L}_{\mathrm{q}}\right)$ | 10 minutes/customer |
| Average time a customer spends in the system $\left(\mathrm{W}_{\mathrm{s}}\right)$ | 7.8976 minutes/customer |
| Average time a customer spends in waiting in the queue $\left(\mathrm{W}_{\mathrm{q}}\right)$ |  |

The results on the above Table 2.2 .5 shows that, the server would be busy $77.55 \%$ of the time and idle for $22.45 \%$ of the time. Also, there are 3.4550 customers in the system on average and the average number of customers in the queue is shown to be 2.6797 customers. Furthermore, the average time a customer spends in the system is 10 minutes/customer and the average time a customer spends in waiting in the queue is 7.8976 minutes/customer.
4.2.6 Computations of Parameters on $18^{\text {th }}$ March, 2018 From
$\mathrm{r}=$ Inter-arrival time $=$ time between two successive arrivals $=60$
$\mathrm{E}[\mathrm{r}]=\frac{60}{21}=2.8571$
$\lambda=$ Mean arrival rate $=\frac{1}{E[r]}=\frac{1}{2.8571}=0.3500$
$\mathrm{s}=$ Mean service time $=33$
$\mathrm{E}[\mathrm{s}]=\frac{33}{15}=2.2000$
$\mu=$ Mean service rate $=\frac{1}{E[s]}=\frac{1}{2.2000}=0.4545$

Utilization factor $(\varrho)=\frac{\lambda}{\mu}=\frac{0.3500}{0.4545}=0.7701$
Probability that the system is idle $(\mathrm{P})=1-\varrho=1-0.7701=0.2299$
Average number of customers in the system $\left(\mathrm{L}_{\mathrm{s}}\right)=\frac{\lambda}{\mu-\lambda}=\frac{0.3500}{0.4545-0.3500}=3.3493$ customers
Average number of customers in the queue $\left(\mathrm{L}_{\mathrm{q}}\right)=\frac{\lambda^{2}}{\mu(\mu-\lambda)}=\frac{0.3500^{2}}{0.4545(0.4545-0.3500)}=2.5792$ customers
Average time a customer spends in the system $\left(\mathrm{W}_{\mathrm{s}}\right)=\frac{1}{\mu-\lambda}=\frac{1}{0.4545-0.3500}=9.5694$ minutes $/$ customer
Average time a customer spends in waiting in the queue $\left(W_{q}\right)=\frac{\lambda}{\mu(\mu-\lambda)}=\frac{0.3500}{0.4545(0.4545-0.3500)}=7.3692$ minutes/customer. The results calculated shows that, the server would be busy $77.01 \%$ of the time and idle $22.99 \%$ of the time. It also shows that, average number of customers in the system is 3.3493 and average number of customers in the queue is 2.5792 . Moreover, the average time a customer spends in the system is 9.5694 minutes and average time a customer spends in the queue is 7.3692 minutes.

Table 4.2.7 Parameter Estimates on 18th march 2018

| INPUT | UNIT MEASURE |
| :---: | :---: |
| Arrival rate ( $\lambda$ ) | 0.3500 |
| Service rate ( $\mu$ ) | 0.4545 |
| Intermediate Calculations |  |
| Average inter- arrival time | 2.8571 |
| Average service time | 2.2000 |
| Performance Measures |  |
| Average server Utilization (rho) | 0.7701 |
| Probability that the service unit is Idle (p) | 0.2299 |
| Average number of customers in the system ( $\mathrm{L}_{\mathrm{s}}$ ) | 3.3493 customers |
| Average number of customers in the queue ( $\mathrm{L}_{\mathrm{q}}$ ) | 2.5792 customers |
| Average time a customer spends in the system ( $\mathrm{W}_{\mathrm{s}}$ ) | 9.5694 minutes/customer |
| Average time a customer spends in waiting in the queue $\left(\mathrm{W}_{\mathrm{q}}\right)$ | $7.3692 \mathrm{minutes} /$ customer |

Source: field survey march, 2018
Table 4.2.8 Parameter Estimates on 19th march, 2018

| INPUT | UNIT OF MEASURE |
| :--- | :--- |
| Arrival rate $(\lambda)$ | 0.3051 |
| Service rate $(\mu)$ <br> Intermediate Calculations <br> Average inter- arrival time | 0.400 |
| Average service time | 3.2778 |
| Performance Measures | 2.5000 |
| Average server Utilization (rho) | 0.7628 |
| Probability that the service unit is Idle (p) | 0.2373 |
| Average number of customers in the system $\left(\mathrm{L}_{\mathrm{s}}\right)$ | 3.2150 customers |
| Average number of customers in the queue $\left(\mathrm{L}_{\mathrm{q}}\right)$ | 2.4522 customers |
| Average time a customer spends in the system (W) | 10.5374 minutes/customer |
| Average time a customer spends in waiting in the queue | 8.0374 minutes/customer |
| $\left(\mathrm{W}_{\mathrm{q}}\right)$ |  |

Source: field survey march, 2018
Table 4.2.6 shows the results of data collected on the 19 th march, 2018. It shows that the server was $76.28 \%$ busy of the time and $23.73 \%$ idle of the time.

Table 4.2.10 Analysis and Presentation of parameter Estimates on 23rd march, 2018

| INPUT | UNIT MEASURE |
| :--- | :--- |
| Arrival rate $(\lambda)$ | 0.3220 |
| Service rate $(\mu)$ | 0.4516 |
| Intermediate Calculations | 3.1053 |
| Average inter- arrival time | 2.2143 |
| Average service time |  |
| Performance Measures | 0.7130 |
| Average server Utilization (rho) | 0.2870 |
| Probability that the service unit is Idle (p) | 2.4846 customers |
| Average number of customers in the system $\left(\mathrm{L}_{\mathrm{s}}\right)$ | 1.7715 customers |
| Average number of customers in the queue $\left(\mathrm{L}_{\mathrm{q}}\right)$ | 7.7160 minutes/customer |
| Average time a customer spends in the system (W) |  |
| Average time a customer spends in waiting in the queue | 5.501 minutes/customer |
| $\left(\mathrm{W}_{\mathrm{q}}\right)$ |  |

Source: field survey march, 2018
Table 4.2.10 above shows that, the system will be busy $71.30 \%$ of the time and $28.70 \%$ idle of the time.

Table 4.2.12 Presentation of Summary of parameter estimates for the various six (6) days period

| INPUTS | DATES |  |  |  |  |  | UNITS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 17/03/18 | 18/03/18 | 19/03/18 | 22/03/18 | 23/03/18 | 24/03/18 |  |
| Total time | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |  |
| $\Lambda$ | 0.3393 | 0.3500 | 0.3051 | 0.3729 | 0.3220 | 0.2807 |  |
| $\mu$ | 0.4375 | 0.4545 | 0.4000 | 0.4516 | 0.4516 | 0.4286 |  |
| Server(s) | 1 | 1 | 1 | 1 | 1 | 1 |  |
| Model | M/M/1 | M/M/1 | M/M/1 | M/M/1 | M/M/1 | M/M/1 |  |
| INTERMEDIATE CALCULATIONS |  |  |  |  |  |  |  |
| Av.Int.Time | 2.9474 | 2.8571 | 3.2778 | 2.6818 | 3.1053 | 3.5625 |  |
| Av.S.Time | 2.2857 | 2.2000 | 2.5000 | 2.2143 | 2.2143 | 2.3333 |  |
| PERFORMANCE MEASURE |  |  |  |  |  |  |  |
| RHO(@) | 0.7755 | 0.7701 | 0.7628 | 0.8257 | 0.7130 | 0.6549 |  |
| P | 0.2245 | 0.2299 | 0.2373 | 0.1743 | 0.2870 | 0.3451 |  |
| $\mathrm{L}_{\mathrm{s}}$ | 3.4552 | 3.3493 | 3.2150 | 4.7382 | 2.4846 | 1.8979 | Customers |
| $\mathrm{L}_{\mathrm{q}}$ | 2.6797 | 2.5792 | 2.4522 | 3.9125 | 1.7717 | 1.2430 | Customers |
| $\mathrm{W}_{\text {s }}$ | 10 | 9.5694 | 10.5374 | 12.7065 | 7.7160 | 6.7613 | Minutes/c |
| $\mathrm{W}_{\mathrm{q}}$ | 7.8976 | 7.3692 | 8.0374 | 10.4921 | 5.5017 | 4.4281 | Minutes/c |

Table 4.2 .5 shows the analysis of the various days data collected. The results show that, $22^{\text {nd }}$ results gave the highest server's busy time of $82.57 \%$ and with the least idle time of $17.43 \%$. It was followed by $17^{\text {th }}$ with the server busy time of $77.55 \%$ and idle time of $22.45 \%, 18^{\text {th }}$ with the busy time of $77.01 \%$ and idle time of $22.99 \%$,
$19^{\text {th }}$ with busy time of $76.28 \%$ and idle time of $23.73 \%, \quad 23^{\text {rd }}$ with service busy time of $71.30 \%$ and idle time of $28.70 \%$ and $24^{\text {th }}$ with the least service busy time of $65.49 \%$ and highest idle time of $34.51 \%$.
Moreover, the results show that, the more the idle time, the lesser the average number of customers in the system and in the queue and the lesser the average time a customer spends in the system and waiting in the queue and vice versa.

Table 4.2.13 Comparison of Utilization Factors for $\mathbf{M} / \mathbf{M} / 1, \mathbf{M} / \mathbf{M} / 2$ and $\mathbf{M} / \mathbf{M} / 3$ for the Various Days.

| DATES | $\mathrm{M} / \mathrm{M} / 1$ | $\mathrm{M} / \mathrm{M} / 2$ | $\mathrm{M} / \mathrm{M} / 3$ |
| :--- | :--- | :--- | :--- |
| 17th March, 2018 | 0.7755 | 0.3878 | 0.2585 |
| 18 th March, 2018 | 0.7701 | 0.3850 | 0.2567 |
| 19th March, 2018 | 0.7628 | 0.3814 | 0.2543 |
| 22 ${ }^{\text {th }}$ March,2018 | 0.8257 | 0.4129 | 0.2752 |
| 23 ${ }^{\text {rd }}$ March,2018 | 0.7130 | 0.3565 | 0.2377 |
| 24th March, 2018 | 0.6549 | 0.3275 | 0.2183 |

Source: field survey march, 2018
From table 4.2.6 above, on 17 th march, the results show $77.55 \%$ busy time for a single server $/$ teller $(\mathrm{m} / \mathrm{m} / 1)$, $38.78 \%$ busy time for two servers ( $\mathrm{m} / \mathrm{m} / 2$ ) and $25.85 \%$ busy time for three servers ( $\mathrm{m} / \mathrm{m} / 3$ ). Also, $18^{\text {th }}$ march results show $77.01 \%$ busy time for a single server, $38.50 \%$ busy time for two servers and $25.67 \%$ busy time for three servers. Moreover, the remaining days follows the same routine. Hence, as the number of servers increases the utilization factor decreases.

Figure 4.2 Comparison of Utilization factors of $m / m / 1, m / m / 2$ and $m / m / 3$ for the various days.


Figure 4.4 above shows the comparison of utilization factors using different types of queuing models. It is shown that, the single server model has the longest bars followed by the two servers' model and the three servers' model with the shortest bars. Indicating that, the less the servers, the higher the utilization factor and vice versa.

## Analysis of Binary Logistics Regression

Atest on customers' satisfaction of the single server in the banking hall of Afram Community Bank Limited using Binary Logistics Regression

## Hypothesis;

$\mathrm{H}_{0}$ : Customers are not satisfied with the single server in the bank
$\mathrm{H}_{1}$ : Customers are satisfied with the single server in the bank

Table 4.3.1 Omnibus Tests of Model Coefficients

|  | Chi-square | Df | Sig. |
| :--- | :---: | :---: | :---: |
| Step | 14.742 | 2 | .001 |
| Block | 14.742 | 2 | .001 |
| Model | 14.742 | 2 | .001 |

The $-2 \log$ likelihood (sometimes called, deviance) has a chi square distribution. The value for result of adding sex (gender) to the model is given in the above table with a value of 0.001 which is less than the conventional significance value of 0.005 . Hence, we would conclude that adding sex (gender) is statistically significant, customers are not satisfied with the single channel queuing model.

Table 4.3.2 Model Summary

| Step | -2 Log likelihood | Cox \& Snell R Square | Nagelkerke R Square |
| :--- | :--- | :--- | :--- |
| 1 | $145.082^{\mathrm{a}}$ | .166 | .222 |

a. Estimation terminated at iteration number 4 because parameter estimates changed by less than .001 .

Table 4.3.3 Variables in the Equation

|  | B | S.E. | Wald | df | Sig. | $\operatorname{Exp}(B)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sex (1) | 1.134 | .425 | 7.107 | 1 | .008 | 3.108 |
| Constant | -1.911 | .523 | 13.337 | 1 | .000 | .148 |

The above shows that, a unit increase in females will lead to 3.108 times they are likely not to be satisfied with the single channel queuing model. If you want the odds that, the females will be satisfied with the single channel queuing model is the inverse of 3.108 .

- Logistics Model
$\log \left(\frac{p_{i}}{1-p_{i}}\right)=\operatorname{logit}\left(p_{i}\right)=\beta_{0}+\beta_{1} x_{i}$
Where; $\mathrm{p}_{\mathrm{i}}=$ the probability of females not satisfied with the single channel queuing model.
The model can be written in terms of odds
$\frac{p_{i}}{1-p_{i}}=e^{\left(\beta_{0}+\beta_{1} x_{i}\right)}$
$p_{i}=\frac{e^{\left(\beta_{0}+\beta_{1} x_{i}\right)}}{1+e^{\left(\beta_{0}+\beta_{1} x_{i}\right)}}$
- Conversely, the probability of females who are satisfied with the single channel queuing model is (1- $\mathrm{P}_{\mathrm{i}}$ )
$\mathrm{P}_{\mathrm{i}}=\frac{e^{(-1.911+1.134(2))}}{1+e^{(-1.911+1.134(2))}}=\frac{1.42903587}{1+1.42903587}=0.5883140252$
$1-P_{i}=1-0.5883140252=0.4116859748$
ODDS $=\frac{\text { probability of success }}{\text { probability of failure }}=\frac{0.5883140252}{0.4116859748} \approx 1.42903587$
- A Unit Increased in Females;
$\mathrm{P}_{\mathrm{i}}=\frac{e^{(-1.911+1.134(3))}}{1+e^{(-1.911+1.134(3))}}=\frac{4.441534834}{1+4.441534834}=0.8162283196$
» $1+\mathrm{P}_{\mathrm{i}}=1-0.8162283196=0.1837716804$
Odds $=\frac{0.8162283196}{0.1837716804}=4.441534832$
ODDS RATIO $=\frac{4.441534832}{1.42903587}=3.108063917 \approx 3.108$
Therefore, per the analysis above, we fail to reject the null hypothesis and conclude that, customers are not satisfied with the single channel queuing model.


## Summary of Findings

The primary objective of this project was to determine how effective the single channel queuing model ( $\mathrm{M} / \mathrm{M} / 1$ ) can be used to analyze the utilization and performance measures and how it relates to customer satisfactions in the Afram Community Bank Limited.
From Table 4.1.2 the findings on the questionnaire data shows that, high service charge, long waiting time and network problems were responded by customers as their major problems faced during services in the Afram Community Bank Limited.

From table 4.2.5 the results on server utilization factors for the various days shows that, $22^{\text {nd }}$ march, 2018 recorded the highest utilization factor of 0.8257 . It was followed by $17^{\text {th }}$ with a utilization factor of 0.7755 , $18^{\text {th }}$ with utilization factor of $0.7701,19^{\text {th }}$ with a utilization factor of $0.7628,23^{\text {rd }}$ with utilization factor of 0.7130 and $24^{\text {th }}$ with the least utilization factor of 0.6549 . The results shows that, as the utilization factors increases, there is a relative increased in the average number of customers in the system, increased in the average number of customers in the queue, increased in the average time a customer spends in the system increased in the average time a customer spends in waiting in the queue.
Table 4.2.6 shows the results of the analysis of different queuing models. The results show that, as the number of servers' increases, the corresponding utilization factors decreases. For instance, the utilization factor of a single server on the $17^{\text {th }}$ was 0.7755 , it decreases to 0.3878 when there are two servers and further decreases to 0.2585 when there are three servers.

A binary Logistics Regression was further used to test on customers satisfactions with the single teller/server in the bank. And the null hypothesis was failed to reject and hence it was concluded that, customers are not satisfied with the single server in the Afram Community Bank Limited (Maame-Krobo Branch).

## Conclusion

The study looked at the application of single channel queuing model $(\mathrm{m} / \mathrm{m} / 1)$ to help reduce the waiting time of customers in the Afram Community Bank Limited to enhance bank services in other to increase customers' satisfaction.
The model was effectively used and the results obtained from the analysis were significantly encouraging. Though customers preferred time was between 6-10 minutes. The highest average waiting time of customers in the system and in the queue was 12.7065 minutes and 10.4921 minutes respectively. Also, the highest average number of
customers in the system and in the queue was 4.7382 customers/minute and 3.9125 customers/minute respectively. In conclusion, customers will remain loyal if and only if their satisfactions are met.

## Recommendations

Based on the summary and conclusion on the findings of this study, the following recommendations have been made to help management improve upon customer satisfaction and also help reduce customers waiting time in the banking hall.

1. Management should improve on the network system in the banking hall of the Maame - Krobo branch of the Afram Community Bank Limited.
2. Management should adopt a two-teller or more system to help reduce the waiting time of customers in the banking hall during services.
3. Management should reduce the service charges on customers such that loyal customers will remain very reliable to the bank.

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